Principle of equivalence and wave-paticle duality in quantum gravity

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February 7, 2008

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PUBLISHED IN: *Memorias del III Taller de la DGFM-SMF, 2000. "Aspectos de Gravitación y Física-Matemática"*, Editadas por N. Bretón, O. Pimentel y J. Socorro.

POST-PUBLICATION COMMENTS: The factor of 8 in Eq. (23) should be replaced by 4. This talk is based on: G. Z. Adunas, E.Rodriguez-Milla, D. V. Ahluwalia, Phys. Lett. B. 485 (2000) 215-223, and D. V. Ahluwalia, Phys. Lett. A (in press), gr-qc/0002005. The post-talk evolution of these ideas is contained in those two publications, and in G. Z. Adunas, E.Rodriguez-Milla, D. V. Ahluwalia, Gen. Rel. Grav. (in press), gr-qc/0006022.

Abstract

This talk presents: (a) A quantum-mechanically induced violation of the principle of equivalence, and (b) Gravitationally-induced modification to the wave particle duality. In this context I note that the agreement between the predictions of general relativity and observations of the energy loss due to gravitational waves emitted by binary pulsars is just as impressive as the agreement between prediction of quantum electrodynamics and the measured value of Lamb shift in atoms. However, general relativity has not yet yielded to a successful quantised theory. There is a widespread belief that the two thories are incompatible at some deep level. The question is [1]: where? Here, I show that the conceptual foundations of the theory of general relativity and quantum mechanics are so rich that they suggest concrete modifications into each other in the interface region. Specifically, I consider quantum states that have no classical counterpart and show that such states must carry an inherent violation of the principle of equivalence. On the other hand, I show that when gravitational effects are incorporated into the quantum measurement process one must induce a gravitationally induced modification to the de Broglie's wave-particle duality. The reported changes into the foundations of the two theories are far from in-principle modifications. These are endowed with serious implications for the understanding of the early universe and, in certain instances, can be explored in terrestrial laboratories.

To establish the thesis outlined in the abstract, first consider a flavor eigenstate of a neutrino as a standard linear superposition of different mass eigenstates:

$$|\nu_\ell\rangle = \sum_{\jmath} \mathcal{F}_{\ell\jmath}\,|m_\jmath\rangle \tag{1}$$

where the flavor index ℓ carries the values e, μ, τ. The mass eigenstates are labeled by $\jmath$ which takes on the values $1, 2, 3$. Finally, the $\mathcal{F}_{\ell\jmath}$ are elements of a 3×3 unitary mixing matrix, a rough form of which may be deciphered from the neutrino-oscillation data (see, e.g., ref. [2]).

Equation (1) makes the non-commutativity of the flavor and mass measurements for neutrinos manifest and thus warns for exercising conceptual caution. Phenomenologically, the theory of mass and flavor eigenstates can be incorporated into the standard model in a generally covariant manner, i.e.,

in strict accordance with the equivalence principle. However, measurement theory does not necessarily respect general covariance for quantum states with no classical analog. Below, I argue how the interplay between measurement and the extended standard model phenomenology of equation (1) yields a subtle violation of the equivalence principle.

Now, for simplicity, I confine to a two flavor space. Let the "flavor" states $|\nu_a\rangle$ and $|\nu_b\rangle$ be the following linear superposition of mass eigenstates $|m_1\rangle$ and $|m_2\rangle$:

$$|\nu_\alpha\rangle = c_\theta|m_1\rangle + s_\theta|m_2\rangle \tag{2}$$

$$|\nu_\beta\rangle = -s_\theta|m_1\rangle + c_\theta|m_2\rangle \tag{3}$$

where $s_\theta = \sin(\theta)$, and $c_\theta = \cos(\theta)$. Without any loss of generality, I assume $m_2 > m_1$. I will first restrict myself to the non-relativistic realm, and then immediately proceed to the relativistic case.

In contrast to classical systems, the very quantum construct that defines the flavor eigenstates does not allow them to carry a definite mass. Therefore, within the orthodox interpretational structure of the theory of quantum measurements, the equality of inertial and gravitational masses looses any operational meaning beyond a flavor-dependent fractional accuracy of the order of:

$$f_\eta := \frac{\sqrt{\langle\nu_\eta|\hat{m}^2|\nu_\eta\rangle - \langle\nu_\eta|\hat{m}|\nu_\eta\rangle^2}}{\langle\nu_\eta|\hat{m}|\nu_\eta\rangle} \tag{4}$$

Here, $\eta = \alpha, \beta$, and $\hat{m}|m_\jmath\rangle = m_j|m_\jmath\rangle$, with $\hat{m}$, the mass operator. On evaluating f_η, I find:

$$f_\alpha = \frac{s_{2\theta}\,\delta m}{2\,(m_1 + s_\theta^2\,\delta m)} \tag{5}$$

$$f_\beta = \frac{s_{2\theta}\,\delta m}{2\,(m_1 + c_\theta^2\,\delta m)} \tag{6}$$

where $\delta m = m_2 - m_1$.

Assuming now that both the mass eigenstates carry the same three momentum, $\vec{p}$, and within the standard framework of the neutrino oscillation phenomenology, I obtain in the relativistic limit:

$$f'_\alpha \simeq \frac{\Delta m^2 c^3 s_{2\theta}}{2\,(2p^2 c + \Delta m^2 c^3 s_\theta^2)} \tag{7}$$

$$f'_\beta \simeq \frac{\Delta m^2 c^3 s_{2\theta}}{2\left(2p^2c + \Delta m^2 c^3 c_\theta^2\right)} \tag{8}$$

Here, $\Delta m^2 = m_2^2 - m_1^2$ is the parameter that enters in the kinematic oscillation length:

$$\lambda_0^{osc} = \left[\frac{2\pi \text{ m}}{1.27}\right]\left[\frac{E}{\text{MeV}}\right]\left[\frac{\text{eV}^2}{\Delta m^2}\right] \tag{9}$$

while f'_η is defined as the ratio $\Delta E_\eta / \langle E_\eta \rangle$ paralleling the definition (4). A non-vanishing

$$\begin{aligned} \Delta f'_{\alpha\beta} &:= f'_\alpha - f'_\beta \simeq \frac{\left(\Delta m^2\right)^2 c^8}{16E^4} s_{4\theta} & (10) \\ &= 6.25 \times 10^{-26} \left[\frac{\left(\Delta m^2\right)^2}{\text{eV}^4}\right]\left[\frac{\text{MeV}^4}{E^4}\right] s_{4\theta} & (11) \end{aligned}$$

introduces an additional length scale into the neutrino-oscillation phenomenology:

$$\lambda_{qVEP}^{osc} = \left[\frac{\pi \text{ km}}{5.07}\right]\left[\frac{10^{-15}}{|\Delta f'_{\alpha\beta}\Phi|}\right]\left[\frac{\text{MeV}}{E}\right] \tag{12}$$

In equation (11) I approximate E by pc. This does not introduce any error to the indicated order in the mass squared difference. In equation (12), Φ is an essentially constant gravitational potential due to the rest of the universe in our immediate vicinity. To distinguish better the classical violation of the equivalence principle (VEP), from the quantum mechanically induced one, I here use the abbreviation qVEP for the latter. The planetary orbits are not changed by Φ because it is essentially gradient-less over the entire solar system. However, a constant Φ gives rise to significant effects in neutrino physics. This is because neutrino oscillations form a flavor oscillation clock, and their evolution is sensitive to Φ. It is precisely this sensitivity that endows them with the gravitational red shift dictated by general relativity [3] (also see [4, 5]). The local cluster of galaxies, referred to as the Great attractor (GA), embeds the solar system in a few parts in 10^{11} constant contribution to Φ. Its value was estimated by Kenyon [6]:

$$\Phi_{GA} \simeq -3 \times 10^{-5} \tag{13}$$

For comparison, the terrestrial and solar gravitational potentials on their respective surfaces are:

$$\Phi_E = -6.95 \times 10^{-10}, \quad \Phi_S = -2.12 \times 10^{-6} \tag{14}$$

The conceptual problems associated with estimating Φ have tempted several authors in the neutrino physics community to identify Φ_{GA} with Φ itself. However, there are no *á priorí* reasons to arbitrarily ignore contributions to Φ from the sources beyond the local cluster of galaxies and to not allow a $\Phi \sim 1$, or even larger.

To explore a possible observability of the qVEP for the long-standing solar neutrino anomaly I combine equations (11) and (12) to obtain:

$$\lambda_{qVEP}^{osc} = 3.16 \times 10^9 \pi \frac{E^3}{(\Delta m^2)^2 \, |s_{4\theta}\Phi|} \text{ km} \tag{15}$$

where it is now implicit that Δm^2 is measured in eV^2, and E is expressed in MeV. ¿From this one readily obtains the ratio:

$$\frac{\lambda_{qVEP}^{osc}}{\lambda_\odot} = 0.66 \times 10^2 \frac{E^3}{(\Delta m^2)^2 \, |s_{4\theta}\Phi|} \tag{16}$$

where $\lambda_\odot \simeq 1.5 \times 10^8$ km is the mean Earth-Sun distance relevant for the neutrino detectors on the Earth.

We now study the result (16) for the solar neutrinos where $0.2 \leq E \leq 20$ MeV. The existing and planned non-solar neutrino oscillations experiments explore $\alpha \leq \Delta m^2 \leq 10^{-4}$, where α is of the order of a few eV^2. The neutrino-oscillation parameter space, when firmly investigated, can be combined with result (16), to check the E^3 dependence of qVEP, and to infer Φ. The presently controversial higher end of the parameter Δm^2 will soon be scrutinized by an experiment under construction at Fermi Lab. in Chicago. The lower end of that parameter space is already under intense investigation at Japan's Super-Kamiokande experiment. The conditions for the observability of qVEP in the solar neutrino oscillation data requires, (a) that the parameters $\{\Delta m^2, \theta, \Phi\}$ are such that the ratio $\lambda_{qVEP}^{osc}/\lambda_\odot$ is of the order of unity, and that (b) the data quality and analysis be such that they allow for the separation of the slow and fast degrees of freedom. The fast degree of freedom comes from the usual kinematically induced oscillation length, λ_0^{osc},

while the slow one is induced by qVEP. Further, qVEP's E^3 dependence can be used to distinguish it from the well-known conjecture on VEP due to Gasperini [7, 8].

The qVEP is not restricted to neutrino physics. The Stanford techniques pioneered by Steven Chu [9] can be exploited to study qVEP by means of atomic states modeled after equation (1). At present, these techniques allow for the relevant gravitationally induced phases to be measured with the remarkable precision of a few parts in 10^9.

Having established the phenomena of qVEP, I now present arguments on the gravitationally modified de Broglie wave particle duality for the interface region under study.

Some years ago I formulated an argument [10] that at the Planck scale quantum measurements alter the local space-time metric in a manner that destroys the commutativity of the position measurements. One consequence of that argument was that non-locality must be an essential part of any attempt to merge the theory of general relativity with quantum mechanics. The non-locality derived in that argument easily extends to measurements of different components of the position vector of a single particle, and modifies the fundamental commutators of the Heisenberg algebra. Efforts in string theories come to the same conclusion in an independent manner. In that context an early reference is the work of Veneziano [11] while a recent one is [12]. Without invoking extended objects, and entirely within the framework of quantum mechanics and the theory of general relativity, Adler and Santiago [13] have found at the same modifications to the uncertainty principle as that obtained by works on extended objects (for directly related works see [14, 15, 16, 17]). A somewhat different argument, based on the existence of an upper bound for acceleration, also results in a similar modification to the uncertainty principle [18]. The mathematical expression of the above results that leads to a gravitationally modified expression for the de Broglie wave particle duality is given by the following modification to the fundamental commutator [19]:

$$[\mathbf{x},\,\mathbf{p}] = i\hbar\left[\mathbf{1} + \epsilon\frac{\lambda_P^2\mathbf{p}^2}{\hbar^2}\right] \tag{17}$$

where $\lambda_P = \sqrt{\hbar G/c^3}$, is the Planck length, and ϵ is some dimensionless number of the order of unity. In what follows, for convenience, I set ϵ equal to

unity. This commutator reproduces the gravitationally modified uncertainty relations. Specifically, one has the new position-momentum uncertainty relation:

$$\Delta x\, \Delta p_x \geq \frac{\hbar}{2}\left[1+\left(\frac{\lambda_P \Delta p_x}{\hbar}\right)^2+\left(\frac{\lambda_P\langle\mathbf{p}\rangle}{\hbar}\right)^2\right] \tag{18}$$

which carries with it the Kempf-Mangano-Mann (KMM, ref.[19]) lower bound on the position uncertainty:

$$\Delta x_{_K} = \lambda_P\left(1+\frac{\lambda_P\langle\mathbf{p}\rangle}{\hbar}\right)^{1/2} \tag{19}$$

Notice that Δx_K has a state dependence via $\langle\mathbf{p}\rangle$. For a state of a vanishing $\langle\mathbf{p}\rangle$, one obtains the absolute minimal distance that can be probed quantum mechanically. Since this lowest bound does not depend on the particle species, it is likely that this points towards some new intrinsic property of the space-time itself.

An important implication of the KMM lower bound, $\Delta x_{_K}$, is that the de Broglie plane waves can no longer represent the physical wave functions, even in principle. Thus the de Broglie wave particle duality must undergo a fundamental conceptual and quantitative change.

In their pioneering work KMM have presented a modification to the de Broglie relation, but they have confined only to the non-relativistic particles (a situation likely to be of little interest in the Planck regime). Here I present the gravitationally modified de Broglie relation without restrictions on the particle's momentum. It is readily seen that the momentum space wave function consistent with the gravitationally modified uncertainty relations (18) reads [19]:

$$\begin{aligned}\psi(p) &= N\left(1+\beta p^2\right)^{-\left[\kappa(\mathbf{p})/4\beta(\Delta p)^2\right]} \\ &\quad\times\exp\left[-i\,\frac{\langle\mathbf{x}\rangle}{\lambda_P}\tan^{-1}\left(\sqrt{\beta}p\right)-\frac{\kappa(\mathbf{p})\langle\mathbf{p}\rangle}{2(\Delta p)^2\sqrt{\beta}}\tan^{-1}\left(\sqrt{\beta}p\right)\right]\end{aligned} \tag{20}$$

where $\kappa(\mathbf{p}) := 1+\beta(\Delta p)^2+\beta\langle\mathbf{p}\rangle^2$, and $\beta := \lambda_P^2/\hbar^2$. N is a normalization factor. This represents an oscillatory function damped by a momentum-

dependent exponential. I identify the oscillation length with the gravitationally modified de Broglie wave length:

$$\lambda = 2\pi \frac{\lambda_P}{\tan^{-1}\left(\sqrt{\beta}p\right)} \tag{21}$$

Introducing $\overline{\lambda}_P := 2\pi\lambda_P$ as the *Planck circumference*; and λ_{dB} as the gravitationally **un**modified de Broglie wave length, $\lambda_{dB} = h/p$, the above expression takes the form:

$$\lambda = \frac{\overline{\lambda}_P}{\tan^{-1}\left(\overline{\lambda}_P/\lambda_{dB}\right)} \begin{cases} \rightarrow \lambda_{dB} & \text{for low energy regime} \\ \rightarrow 4\lambda_P & \text{for Planck regime} \end{cases} \tag{22}$$

In addition, for the specific non-relativistic regime considered by Kempf *et al.* [19], λ reproduces their equation (44). This justifies the interpretation of the oscillatory length associated with KMM's $\psi(p)$ as the gravitationally modified de Broglie wavelength.

It is worth repeating that in the Planck realm, the wavelength λ asymptotically approaches the constant value $4\lambda_P$ that is now universal for all particle species. As a consequence of this universality, a new type of coherence is likely to emerge in the early universe whose significance for the large-scale uniformity of the universe has already been mentioned.

As an illustrative example, to the lowest order in λ_P, the effect of the modification (18) on the ground state level of the hydrogen atom results in the following modified uncertainty principle estimate for the ground state of an electron in an H-atom:

$$\left(E_0\right)_g \simeq -\frac{me^4}{2\hbar^2}\left[1 - \frac{8m\lambda_P^2}{\hbar^2}\left(\frac{me^4}{2\hbar^2}\right)\right] \tag{23}$$

Identifying:

$$E_0 = -\frac{me^4}{2\hbar^2} \tag{24}$$

with the ground state level of the hydrogen atom without incorporating the gravitationally-induced correction to the uncertainty relation, one immediately notices that the effect of the gravitationally induced modification is to

reduce the magnitude of the ionization energy by 5×10^{-48} eV. This suggests that a space-time endowed with the KMM bound is in some sense a heat bath as it decreases the energy required to disassociate the H-atom.

If the effects of the gravitationally induced modification to the de Broglie wave particle duality are negligible for low energy, their relevance can hardly be overestimated at the Planck-scale induced phenomena. Apart from the coherence that is predicted to be present in the early universe, there are already speculations that anomalous events around 10^{20} eV cosmic rays may be pointing towards a violation of the Lorentz symmetry [20, 21]. This violation is also independently expected from the present study and can be systematically studied as new cosmic ray experiments yield data at still higher energies approaching the Planck scale. In this context it is important to note that the argument that Planck energy cosmic rays are forbidden by the Greisen-Zatsepin-Kuzmin (GZK) cutoff is no longer valid as the recent cosmic ray experiments have amply demonstrated.

The above discussion makes it clear that the conceptual foundations of the theory of general relativity and quantum mechanics are so rich that they suggest concrete modifications into each other in the interface region.

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Acknowledgements. This was my first invited talk in Mexico after I moved to Mexico as a faculty member at Zacatecas. I extend my thanks for the invitation, and for the opportunity to meet a vibrant community full of new ideas and hopes. My particular thanks go to the leadership of Mexican Physics for making my coming to Mexico possible, and so rewarding. My editorial responsibilities, and efforts involved in settling (not to mention applying for the CONACyT project and for the SNI membership), proved to be so overwhelming for me that so far my Spanish classes have taken a back seat. For that my apologies. I hope in some small way I compensate that failing in other ways. This work was supported by CONACyT (Mexico) and good cheer of the desert mountain. Gracias!